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A Summary of the Research Progress on Grant AFOSR-76-3017

From July 11, 1976 to Aug. 31, 1979

Introduction

The research activities supported by the grant AFOSR-76-3017 can be broadly classified into the following areas:

(1) Communication Networks

(2) Variable Length Codes

(3) Digital Systems and Computer Algorithms

(4) Topics in Pure and Applied Graph Theory.

We will very briefly describe the successful results leading to publication in each of these areas.

(1) Communication Networks

Hakimi defined the p-medians and p-centers of a network about 15 years ago. Since then a field centered about these notions has evolved called "location theory." In a more recent paper, Hakimi, Schmeichel and Pierce [1] showed that Hakimi's original 1-center algorithm can be refined to obtain algorithms of complexities $O(n^4 \log n)$ and $O(n^3 \log n)$ for finding a 1-center of vertex weighted network and a 1-center of a vertex unweighted network, respectively. This leads to an $O(n^3 \log n)$ algorithm for finding a least diameter tree of a graph. Kariv and Hakimi [2] improved the results in [1] and also showed that then 1-center problem is NP-complete. Kariv and Hakimi [3] then showed that p-median problem is in general NP-complete; but presented an $O(n^2p^2)$ algorithm for finding a p-median of a tree. Hakimi was invited to present lectures at ORSA/TIMS on the p-centers in networks [4] and at International Symposium on Locational Decisions on the network location theory [5]. Finally Megiddo and Hakimi [6] studied the search number of a graph. This problem is related to the problem how to capture a mobile fugitive in a highway network with the least number of searchers.

(2) Variable Length Codes

It is known that in constructing an optimum (least average cost of transmission) variable length code one can look for an optimum prefix code and be assured that this code is in fact the best possible (uniquely decipherable) code so long as the cost of transmission of each character is the same as the cost of transmission of any other character. However, if this cost assumption cannot be nat mode, then it is not known whether a globally optimum code could be a prefix code or not. This is, however, often conjectured but never proved. Ntafos and Hakimi [7] give further evidence for the validity of this conjecture by showing that it is true if there are only two code word lengths.

(3) Digital Systems and Computer Algorithms

This area constitutes our greatest effort and may be subdivided into two parts: (a) Fault Analysis and Software Validation and (b) Computer Algorithms and Complexity

(a) Fault Analysis and Software Validation.

This author and his colleagues have made significant contribution to the theory of t-diagnosable systems. This theory was initiated by the work of Preparata, Metze and Chien and has since become a significant approach to fault analysis particularly in those digital systems which consist of a network of computers. This author's work has paved the way for many generalizations and variations. Two lectures in these subjects were presented at major scientific meeting [8,9]. We are at present working on certain generalizations of this subject and also are demonstrating the usefulness of t-diagnosable systems in the design of fault-tolerant computing systems.

This work is in progress and expected to be completed in a near future.

A feasible approach to software validation involves covering a digraph representing the flow chart of a computer program, preferably, by a least number of paths (chains). Dilworth's Theorem was generalized and efficient algorithms were suggested for the above covering problem by Ntafos and Hakimi [10]. In the same paper, the authors show that to model the interaction among the program segments one need to introduce the notions of "must pair," "impossible pair," etc. Such considerations, however, immediately makes the resulting cover problems NP-complete. Ntafos and Hakimi [11] later applied their theory to structured programs. The result of their study was that although structured programs are easier to design and to debug and possibly to test; but ultimately the problems arising in program testing are nearly all NP-complete even for structured programs.

(b) Computer Algorithms and Complexity

Although it is known that scheduling problems are difficult even on a single machine. In fact, it is known that: given a set of independent tasks $\{T_1, T_2, \ldots, T_n\}$ with T_i having a given release time r_i and deadline d_i and processing time p_i; the question, can these tasks be processed on a single machine, is NP-complete. However, many restricted versions of the above problem are not. More recently because of the availability of inexpensive hardware the notion of parallel processing of tasks has become a serious field of interest. Nakajima and Hakimi [12,13] in two papers have shown that even in the restricted cases when the single machine task scheduling problem is tractable, the corresponding multi-machine versions often are NP-complete. Also they demonstrate that there are many interesting multi-machine problems that are tractable and sometimes very efficiently solvable.

Ntafos and Hakimi [14] attempted to prove a conjecture by Berlekamp, McEliece and Van Tilborg that the problem finding the Hamming distance of a binary code is NP-complete. Their results shed light on the problem and it does strongly indicate that the conjecture is true.

A heuristic approach to the very important NP-complete set covering problem was considered by Chwa and Hakimi [15].

4. Topics in Pure and Applied Graph Theory

In a series of papers, Hakimi and Schmeichel [16,17,18] studied various properties of planar graphs, among them the connectivity of such graphs, the number of cycles of length k in such graphs. Earlier the same authors had studied the degree sequences of planar graphs [19] and traceable graph [20]. A tutorial paper [21] by the same authors presents a summary of the theory of graphical degree sequences. A generalization of such a theory was considered by Patrinos and Hakimi [22]. Finally, Hakimi, Schmeichel and Thomassen studied the number of Hamiltonian cycles in a maximal planar graph [23].

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